A Survey of Models for Interindustry Economics

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I. Economic Models

One of the main characteristics of the economy of a modern, developed country is its variety. The economic situation at a certain period cannot be described by a few, but only by a large number of characteristics. A historic, simultaneous study of some selected economical characteristics usually indicates certain regularities within the economy. This fact leads to the basic postulate of all economic research: There exist some economic "laws" determining the behaviour of the economy. By simplifying and idealizing the observed regularities, it may be possible to organize a comprehensive system representing the assumed "laws."

Due to practical causes it may be necessary to restrict economic research to a limited number of characteristics. Theories about an uncomplete number of interrelated characteristics must be supported by supplementary assumptions about the characteristics determined outside the scope of the study. If a theory is used to predict an unobserved situation, this

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An inductive process will be called a forecast. A forecast is said to be conditional if it relies on supplementary assumptions about the characteristics.

An economy is to a large extent described by quantitative characteristics. It is therefore convenient to give the theories a mathematical formulation. Quantitatively measured characteristics will be called variables and theories about variables will be called models.

The observed economic regularities are simplified in an algebraic model, the structure of which depends on the values of the structural parameters. From a mathematical point of view all forecasts deduced from such a model subject to any set of structural parameters are correct, but from an economic point of view they are, however, only deductions from a mathematical system of relations. An economically useful model must give forecasts with a predescribed degree of accuracy.

The specifications of the structural parameters must always build on earlier observations of the variables included in the study. Regarding each simultaneous set of observations as independent multidimensional random variables and assuming that they all are produced by the same probability mechanism under certain different permitted conditions, it will often be possible to find numerical estimates of the parameters and to derive some measure of their reliability. The model with specified parameters can in this case be regarded as a hypothesis about the characteristics of a conditional probability distribution and on basis of earlier observations this hypothesis may be tested. A forecast from such a probability model will then be a statement about the conditional distribution of an unobserved variable.

In this survey we will not be concerned about methods of specifying the structural parameters, but only discuss the algebraic models used in the interindustry approach to economic research.

II. The Interindustry Flow Matrix

The different interindustry models described in this survey intend to give a quantitative explanation of the flows of economic objects among a set of sectors or industries. The models are therefore partial and cannot be used for solving problems outside the certain class for which they are constructed. The flows studied in an interindustry model may be of different kinds and one particular flow may be measured in different ways. The discussion here will be limited to models explaining flows of commodities

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and services among a specified set of sectors by a system of linear relations and certain outside conditions.

Even though this is a great limitation, it includes a large number of possible models. Common for all of them is that they require some historic observations of the flows if the structural parameters shall be identified. A systematic and very useful presentation of these historic observations is a flow table or matrix for each time period covered by the observations. A flow matrix can be interpreted as a system of double entry book-keeping which shows the magnitude of the flows in and out of each sector during the time studied\(^1\).

Symbolically, a flow matrix is denoted by

\[
X_{ij} = \begin{bmatrix}
x_{11} & \cdots & x_{1m} \\
\vdots & \ddots & \vdots \\
x_{m1} & \cdots & x_{mm}
\end{bmatrix}
\] (II. 1)

where an element \(x_{ij}\) represents the flow from sector \(i\) to sector \(j\). Further assuming additivity

\[
\sum_{j=1}^{m} x_{ij} \quad \text{and} \quad \sum_{i=1}^{m} x_{ij}
\]

represent respectively the total flow from sector \(i\) and the total flow to sector \(j\). \(x_{ii}\) represents the intrasectoral flows in sector \(i\).

As well as being basis for model constructions, a flow matrix has its own value because it reveals inconsistencies in existing statistics and also is a very comprehensive representation of a large mass of statistical information\(^2\).

The form of a matrix which is organized only to give a systematic description of flows, will depend mainly on the existing statistics and may differ from a matrix compiled especially for model construction. If the purpose is to create a basis for an interindustry model, consideration must be taken also to factors as type of model, required specification of the forecast etc. On this stage it may also be necessary to decide whether a model with satisfactory properties can be constructed and eventually for which cost.

In defining a flow matrix there are three main questions to be considered:

(1) The time period. Flows cannot be measured at a certain point, but only between two points of time. As a year is a period to which most of the statistics are referred, it seems usually to be the most opportune time unit. One year may, however, be better than another and is therefore chosen as the matrix year. Certain models require that the matrix year must be a recent one. If more than one matrix is wanted in order to improve the reliability of the model, it may be impossible to find enough recent years which are satisfactory covered from a statistical point of view. One possibility may be to choose a shorter time unit and try to break down the statistical informations to these units. This may not be more difficult than the break down in sectors.

(2) The definition of sectors. To a certain extent, this question is already answered by the required specification of the forecasts. A sector can be defined in many different ways, but in an interindustry study it will usually be defined as all activities having certain specified characteristics in common.

Interindustry models will often rely on assumptions about internal homogeneity in sectors and about substitutable flows. In order to obtain a system of sectors which are internally, sufficiently homogeneous, it may be necessary to make a further break down in sectors than the specification of the forecasts requires. The break down process is, however, limited by several factors. First, the existing statistics are serious limitations, although they can be supplemented by special surveys. But a fine break down introduces a new problem known as the problem of secondary products. This problem arises because the statistical unit on which all information is based, performs activities which according to a fine break down belong to more than one sector. The statistical information about a unit is usually indivisible and the problem of how to classify the unit arises. Third, the specification of sectors satisfying the assumption about substitutable flows probably conflicts more with a fine break down for the purpose of homogeneity than with a less fine break down. Important is also that the marginal cost of a fine break down is in general increasing, a fact which in practice perhaps will be the real limitational factor.

Conclusive for the definitions of the sectors are therefore: (a) The specification of the required forecasts, (b) The type of the model which will be constructed, (c) The existing statistical resources, (d) The homogeneity of the activities, and (e) The budget.

(3) The measurement of flows. The individual items in a flow can often be measured in quantitative terms by physical measures except for several kinds of services. But even in a very homogeneous object flow, there will frequently be used different physical measures. An interindustry matrix presentation requires, however, that each item is additive. This aggregation
problem can be solved by defining a set of valuation or weighting coefficients which can transform the unequally measured items into items measured by a common measure.

Any set of valuation coefficients can be used, but as soon as one particular set is chosen, this implies that the whole model will operate in terms of this set.

From a practical point of view, the most convenient kind of sets is the price set referring to the matrix period. What price set should be used, e.g. producers' or purchasers' value set, will depend on the existing statistics and the type of the model. The use of a price set allows some formal control of the matrix which may be very useful. In an economy the flows from one sector may be defined such that they are equal in value to the flows to the same sector minus a residual flow. Let all the negative residual flows be regarded as flows out of a residual sector and all positive residual flows be flows to the same residual sector. The system or the matrix can then be defined such that we will have the following controls:

\[
\sum_{j=1}^{m} x_{ji} = \sum_{j=1}^{m} x_{ij} \quad (i = 1 \ldots m) \quad (II. 2)
\]

and

\[
\sum_{i=1}^{m} \sum_{j=1}^{m} x_{ji} = \sum_{i=1}^{m} \sum_{j=1}^{m} x_{ij} \quad (II. 3)
\]

Besides these more definitional problems, there will be a great number of practical problems in connection with a matrix organization. These problems will however, to a large extent depend on the special conditions in the different countries.

### III. Static Input-Output Models

#### i. Static Leontief Input-Output Models

(a) The static type of the Leontief input-output model is the most frequently used for interindustry analysis. The fundamental assumptions are that the economy can be divided into a finite number of sectors and that there exist certain interrelated flows among the sectors.

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1 Note the different points of view in the two above papers.
The last assumption is based on the general observation that the output of an activity depends on the amount of input of certain factors. Characteristic for the Leontief approach is the way these relations are specified. It is assumed that there exist a fixed-proportion relation between each input flow and the output. This means constant returns to scale and a fixed relative combination between any pair of inputs to the same sector independent of the scale of production or activity. From the view of a classical production theorist, this assumption may seem to be a too drastic simplification of the real world. How drastic it is cannot be determined before the model is tested. As a justification for this simplification, it is said that subject to a fine break down in industries the relations will be of technical nature and technical relations are often found to be of the proportional type. As the models are static they don’t say whether it is input which determines output or opposite. The relations may therefore also be interpreted as demand for input as determined by output.

The models described in this section will also include one constructed on the more general linearity assumption. This assumption may in some cases be more realistic than the proportionality assumption. On the other hand, the identification of the structural parameters in such a model requires more information.

The basis for the following models are the flow matrices as defined in the previous section.

2. The Closed Leontief Model

Let $X_{ij}$ denote the square $m$-order flow matrix. Then we have the following book-keeping relation for the output distributions:

$$X_{ij} \cdot I = X_i \quad (III. 1)$$

where $I$ is the $m$-order unity column vector and $X_i$ is the output column vector of the same order.

On basis of technical considerations we assume that all inputs to a sector are proportionally related to the output of the same sector. This assumption implies, for instance, that the flows to the residual sector also are proportionally related to the total flow out of this sector. The reliability of such an assumption may of course be discussed. It is also usual to define households as a special sector. The above assumption then says that the flows to the households are proportionally related to the flow out or the services rendered by the sector, a statement which is likely to break down.

Let the assumption be denoted by:

$$x_{ij} = a_{ij} \cdot x_i \quad (i, j = 1 \ldots m) \quad (III. 2)$$

where $x_i$ is the output of sector $i$. 
The square matrix

\[
A = \begin{bmatrix}
  a_{11} & \cdots & a_{1m} \\
  \vdots & \ddots & \vdots \\
  a_{m1} & \cdots & a_{mm}
\end{bmatrix}
\]  

(III. 3)

is called the structural matrix and contains all the structural parameters in the model. This matrix has some important properties which are useful in connection with forecasting. Because of (II. 2—3) and (III. 2) the sum of any column in the matrix is equal to unity. The elements in the matrix are also usually assumed to be non-negative.

(III. 1—2) form the algebraic model. Without knowing anything about the numerical values of the elements of A, the model has little value for forecasting problems. It will, however, be useful to study the mathematical solution to certain forecasting problems.

We may, for instance, be especially interested in the production or the activity levels represented by the vector \( \mathbf{x}_i \). By eliminating the elements of \( x_{ij} \) we get the system:

\[
A \cdot \mathbf{x}_i = \mathbf{x}_i
\]  

(III. 4)

By subtracting \( \mathbf{x}_i \) on each side and changing all signs, we get

\[
\mathbf{x}_i - A \cdot \mathbf{x}_i = [I' - A] \cdot \mathbf{x}_i = 0
\]  

(III. 5)

where \( I' \) denotes the diagonal unity matrix.

The matrix

\[
[I' - A]
\]  

(III. 6)

is called the Leontief matrix. If all the elements in A is non-negative, the Leontief matrix consists of zero-sum column and diagonal elements which are non-negative and dominant. These properties seem to be very important in solving computational problems by approximative methods.

The system (III. 5) forms a linear homogeneous m-order system in the variables included in the vector \( \mathbf{x}_i \). If (III. 6) is a non-singular matrix, this means that the only solution consistent with the system is the solution of zero outputs.

This is obviously not a realistic solution in economic sense. In order to get a more realistic solution, we will assume that the rank of the Leontief matrix is less than m. This means that one or more of the relations are dependent of the other.

Let, for instance, the last relation in (III. 5) be the relation dependent of one of the other and omit it. The rank of the matrix was in this case of

order $m-1$. The system is now reduced to $m-1 = n$ homogeneous relations in the $m$ variables. This model can only give solutions to the relative values of the variables expressed in terms of one of them.

Let the variables be measured in terms of the output of sector $m$. The solution can then be given as:

$$\frac{1}{x_m} \cdot X_{i(n)} = \left[ I - A \right]^{-1} \cdot [A_{i(n)}]$$  \hspace{1cm} (III. 7)

where $X_{i(n)}$ is the $n$-th order column output vector, $[I - A]^{-1}$ is the reciprocal of the principal minor of (III. 6) when the last row and column are omitted, and $A_{i(n)}$ is the column vector of the first $n$ elements in the last column of (III. 6).

This solution subject to certain conditions about the matrix and the model solution states that there always must be a given proportion between any two output levels. The model does not say anything about the absolute levels of outputs.

How realistic our model is depends on the assumptions in (III. 2). Intuitively we may feel that some of the assumptions could be dropped or changed.

3. The Open Leontief Model

This model is of special interest because it was applied in U. S. Bureau of Labor Statistics' large scale interindustry study and is also frequently used in other studies. The model aims at an explanation of the interrelations between output on the one side and a set of autonomous inputs on the other side. The autonomous input and the autonomous sector are often called respectively the final demand and the final demand sector.

Let sector $m$ be the autonomous sector. What output levels are consistent with alternative sets of demand from this sector? A model which formally gives an answer to this question is the following:

Let as before

$$X_{ij} \cdot I = X_i$$  \hspace{1cm} (III. 8)

Suppose that there exist fixed-proportion technical relations between input and output for all the non-autonomous sectors, but no such assumptions are made for the autonomous sector. The input to this sector is assumed determined by some outside mechanism.

We have

$$x_{ij} = a_{ij} \cdot x_j$$  \hspace{1cm} (i = 1, \ldots, m) \hspace{1cm} (III. 9)$$
We define the corresponding structural matrix as

\[ A = \begin{bmatrix}
  a_{11} & \cdots & a_{1n}\cdot 0 \\
  \cdots & \cdots & \cdots \\
  a_{m1} & \cdots & a_{mn}\cdot 0
\end{bmatrix} \]  

Eliminating all the \( x_{ij} \) (\( i = 1, \ldots, m \), \( j = 1, \ldots, n \)), we get \( m \) linear non-homogeneous relations in the \( m \) variables \( x_i \) (\( i = 1, \ldots, m \)):

\[ A \cdot X + X_m = X_i \]  

where \( X_m \) is a column vector representing the autonomous input to the autonomous sector from all the delivering sectors. The system can be transformed to:

\[ (I' - A) X_i = X_m \]  

which has the following formal solution of \( X_i \) expressing the elements of activity in the system

\[ X_i = (I' - A)^{-1} \cdot X_m \]  

The solution of the input matrix is easily obtained by transforming the vector in (III.13) to a diagonal matrix in which all non-diagonal elements are zero and premultiplying by \( A \):

\[ X = A \cdot \left[(I' - A)^{-1} \cdot X_m\right]' \]  

where \([\cdot]'\) denotes a diagonal matrix in which all non-diagonal elements are equal to zero. The input matrix has the elements:

\[ X = \begin{bmatrix}
  x_{11} & \cdots & x_{1n} \\
  \cdots & \cdots & \cdots \\
  x_{m1} & \cdots & x_{mn}
\end{bmatrix} \]  

The limitation of this model type must be born in mind. It is constructed for special purposes and is therefore not universal. Questions about the determination of the autonomous demand can, for instance, not be solved by this type of models. The model is also a static equilibrium model and permits no considerations to what is happening outside the equilibrium position or how long time the economy eventually needs to adjust itself to changes in final demand. Such problems must be studied by other models which are constructed for these purposes. This model gives only a solution to the problem of which output levels are consistent with a certain given demand.
4. An Input-Output Model
Constructed on Linearity Assumptions

As mentioned above, the assumptions in (III.9) may in some cases be made more realistic. In the classical theory of production it is usually assumed that some advantages are obtained by large scale production. The proportionality assumption, however, implies that the return per unit is constant and independent of the scale of production. By introducing the linearity assumptions instead of the proportionality assumptions, we obtain a more general model which permits changing return per unit of input with respect to changes in scale of production:

\[ x_{ij} = a_{ij} \cdot x_j + b_{ij} \quad (i = 1, \ldots, m) \quad (j = 1, \ldots, n) \]  

(III.16)

In addition to the structural matrix (III.10), this model has also another structural matrix:

\[
B = \begin{bmatrix}
    b_{11} & \cdots & b_{1n'0} \\
    \cdots & \cdots & \cdots \\
    b_{m1} & \cdots & b_{mn'0}
\end{bmatrix}
\]  

(III.17)

Corresponding to (III.12) we now get

\[
[I' - A] \cdot X_i = X_m + B_i
\]  

(III.18)

where

\[
B_i = B \cdot I
\]

The solution of the model is given by the output vector

\[
X_i = [I' - A]^{-1} \cdot [X_m + B_i]
\]  

(III.19)

The solution of the input matrix is as above found by transforming (III.19) to a diagonal matrix and premultiplying by A and adding B:

\[
X = A \cdot [(I' - A)^{-1} \cdot [X_m + B_i]]' + B
\]

where the symbol []' denotes a diagonal matrix in which all non-diagonal elements are equal to zero.

5. The Autonomous Sector

The assumptions concerning the input-output relations are to a large extent based on considerations of technical nature. In an economy there will always be activities which are not technical and for which the relations are not so autonomous. Going through a national economy, there are especially five sectors for which the assumptions may be rather irrelevant. These sectors are: (1) Inventories, (2) Capital formation, (3) Foreign trade, (4) Governmental activities, and (5) Households.
For these sectors the balance condition (II.2) is not always realistic. Therefore, instead of having the balance condition (II.2) for each of them, they are regarded as one autonomous sector for which the balance condition holds.

IV. Dynamic Input-Output Models

1. Dynamic Interindustry Models

A model is here defined as a dynamic model if at least one of the variables are connected to different periods or points of time in such a way that it is impossible to transform the model into an equivalent static model where all variables are associated to the same period or the same point of time.

In the static input-output models considered, all variables could be regarded as flows. One of the simplest dynamic approaches is to introduce relations between flows and stocks. A flow can always be considered as the change in a stock from the beginning to the end of a period. If a relation is imposed between the flow in one period and the stock corresponding to the cumulative value of the same flow at some point of time, we get a dynamic relation.

Other dynamic elements which may especially have a certain influence in models constructed on a short time unit are, for instance, the lags between the output and the inputs due to the time required in processing the inputs. Another example is the dynamic element due to the time between observation and action. For models constructed on a time unit equal to a year or more, the capital formation and the change in technology are probably the most important. Unfortunately both these variables are difficult to measure.

A dynamic model is more general than a static. Any static model can be considered as a specialization of a more general dynamic model. As well as forecasting a situation in a certain period in the future, a dynamic model also provides information about how this situation will be obtained. On the other hand it also requires more \textit{a priori} information.

2. An Investment Model

Let us assume that the output of each sector in any period may be regarded as composed of non-capital and capital commodities, and that the inputs to the autonomous sector are non-capital.

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Denoting the flows respectively by \(x_{ij} (i, j = \ldots m)\) and \(y_{ij} (i, j = \ldots n)\), we establish the following book-keeping relations:

\[
[X_{ij}(t) + Y(t)] \cdot I = X_i(t)
\]  
(IV. 1)

where \(X_{ij}(t)\) is the square \(m\)-order input matrix for non-capital commodities in the period \(t\), and \(Y(t)\) is the investment matrix for the period \(t\) defined by

\[
Y = \begin{bmatrix}
y_{11} & \cdots & y_{1n} \\
\cdots & \cdots & \cdots \\
y_{n1} & \cdots & y_{nn}
\end{bmatrix}
\]

We assume as before that there exist fixed-proportion relations between input of non-capital commodities and services, and output, i.e. between inputs as rawmaterial, labor etc., and output:

\[
x_{ij}(t) = a_{ij} \cdot x_j(t) \quad \left( i = \ldots m \right) \quad \left( j = \ldots n \right)
\]  
(IV. 2)

Let the autonomous sector \(m\) be defined such that it also includes capital depreciation. The matrix \(Y\) will then represent net investment.

Let the capital in sector \(j\) received from sector \(i\), at the end of the period \(t\) be defined as:

\[
k_{ij}(t) = y_{ij}(t) + k_{ij}(t-1)
\]  
(IV. 3)

We will assume that the output in the period \(t\) will depend proportionally on the different kinds of capital equipment in the sector at the end of the previous period. If the capital stocks are used in the productive processes, this assumption is realistic. In a contracting economy there will, however, be large amounts of unused capital which of course will not affect the output technically.

Let the above assumption be denoted by:

\[
k_{ij}(t) = n_{ij} \cdot x_j(t+1) \quad \left( i = \ldots n \right) \quad \left( j = \ldots n \right)
\]  
(IV. 4)

The structural parameters \(n_{ij}\) can be symbolically denoted by the capital coefficient matrix:

\[
N = \begin{bmatrix}
n_{11} & \cdots & n_{1n} \\
\cdots & \cdots & \cdots \\
n_{n1} & \cdots & n_{nn}
\end{bmatrix}
\]  
(IV. 5)
To identify the elements of this matrix we need besides the flow matrix also a capital stock matrix:

\[
K = \begin{bmatrix}
k_{11} & \cdots & k_{1n} \\
\vdots & \ddots & \vdots \\
k_{n1} & \cdots & k_{nn}
\end{bmatrix}
\]  

(IV. 6)

By eliminating the elements of the flow matrix \(X\) and the investment matrix \(Y\) from (IV. 1—2—3—4), we get a system of \(n\) linear difference equations and one static linear relation:

\[
[I' - A + N] \cdot X_i(t) - N \cdot X_i(t+1) = X_m
\]  

(IV. 7)

among the variables \(X_i(t)\) and \(X_i(t+1)\). After eliminating the static relation the system can formally be transformed to non-homogeneous \(n\)-order difference equations in any of the variables\(^1\). These equations will be of the form:

\[
c_n x(t+n) + \cdots + c_0 x(t) = c
\]

The solution of these equations may be described by:

\[
X_i(t) = C \cdot R(t) + C_0
\]  

(IV. 8)

where

\[
R(t) = \begin{bmatrix}
\ddots & \vdots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \vdots \\
\ddots & \ddots & \ddots & \ddots
\end{bmatrix}
\]

is the column vector of the atmost \(n\) different roots in the determinant

\[
D(r) = \| I' - A + (r - r) \cdot N \| = 0
\]  

(IV. 9)

while \(C\) and \(C_0\) in (IV. 8) are respectively a square matrix and a column vector both depending on \(X_m\), \(A\) and \(N\). The elements of \(C\) will also depend on the initial values \(X_i(0)\). In any dynamic model it will be necessary to specify the starting point because a part of the task is to explain the development from this point to another.

The solution of the input variables in the matrix \(X\) is now found by combining (IV. 1—2):

\[
X(t) = A \cdot [C \cdot R(t) + C_0]'
\]  

(IV. 10)

where as before the symbol \([\cdot]'\) denotes the diagonal matrix with diagonal elements given by the elements in the vector (IV. 8).

In the same way by combining (IV. 4—8), the capital stocks at the end of the $t$—th period may be expressed by:

$$K(t) = N \cdot [C \cdot R(t + 1) + C_0]' \quad (IV. 11)$$

and consequently we get the following solution for the investment matrix:

$$Y(t) = N \cdot [C \cdot R(t + 1) - C \cdot R(t)]' \quad (IV. 12)$$

The dynamic model introduces several new problems as, for instance, how to calculate the characteristic roots of (IV. 9). On the other hand, it can solve far more interesting problems than a static model.

3. A Model with Lag Between Output and Inputs

All kinds of production or other economic activity take some time. A simple dynamic interindustry model can be obtained by assuming that all processes take a time corresponding to the matrix time unit, i.e. the output depends on the inputs in the previous period.

We will assume that this dependence is of the same kind as assumed in the open Leontief model:

$$x_{ij}(t) = a_{ij} \cdot x_i(t + 1) \quad (IV. 13)$$

As before the following book-keeping relations are supposed to hold:

$$X_i(t - 1) = X_i(t) \quad (IV. 14)$$

Eliminating all the $x_{ij}(t)$ by (IV. 13) we get $m$ linear non-homogeneous difference equations:

$$A \cdot X_i(t + 1) + X_m(t) = X_i(t) \quad (IV. 15)$$

where as before $A$ denotes the structural matrix.

The system can be transformed to a non-homogeneous difference equation in any of the variables. The general solution of the vector of outputs is of the form (IV. 8).

The matrix $A$ of this model does not possess the property of having zero column sums because the inputs are related to outputs of another period.

V. Other Models for Interindustry Economics

It has already been mentioned that the models described in this paper only are a small sample of possible interindustry models. The last dynamic model may, for instance, be made more realistic by assuming that the processing time differs from one sector to another, or by using linear rather than proportional relationships between input and output.
The discussion has also been restricted to linear or proportional assumptions about the input-output relations. This restriction excludes of course many possible interesting and in some sense more realistic static as well as dynamic models for interindustry economics.

Zusammenfassung: Ein Überblick über Modelle zur Untersuchung der wirtschaftlichen Verflechtung der Industrie. — In der vorliegenden Abhandlung behandelt der Verfasser die algebraischen Modelle, die bei Untersuchung der interindustriellen Beziehungen verwendet werden. Dadurch soll eine quantitative Erklärung der Ströme von Gütern und Dienstleistungen zwischen einzelnen Wirtschaftssektoren oder Industrien gegeben werden. Trotz dieser Beschränkung ergibt das eine große Zahl möglicher Modelle. In Form einer Matrix kann der Güterstrom in bzw. aus einem der verschiedenen Sektoren der Wirtschaft oder der Industrie dargestellt werden. Der Verfasser diskutiert die drei Hauptfragen, die sich aus dieser Matrizenrechnung ergeben, nämlich die Festlegung der Zeitperiode, die Abgrenzung der einzelnen Sektoren und die Messung der Güterströme. Im Anschluß daran behandelt er zunächst die statischen Input-Output-Modelle, und zwar besonders das statische Modell von Leontief, das am häufigsten für die Analyse der interindustriellen Beziehungen gebräuchlich wird, in Form des geschlossenen und offenen Modells. Es folgt die Erörterung dynamischer Modelle der interindustriellen Beziehungen, die allgemeinerer Natur sind als die statischen Modelle und die auch Prognosen über die wirtschaftliche Entwicklung ermöglichen. Der Verfasser behandelt zwei Arten dieser dynamischen Modelle ausführlicher, nämlich ein Investitionsmodell und ein Modell mit einem »lag« zwischen Output und Input.

Résumé: Aperçu des modèles qui servent à éclaircir l'enlacement économique de l'industrie. — Dans l'étude que voici l'auteur s'occupe des modèles algébriques, dont on se sert pour étudier les relations interindustrielles. Ils sont destinés à donner une explication quantitative des courants de marchandises et de services, qui passent d'une industrie à une autre et d'un secteur économique à un autre. Malgré cette restriction cela donne un grand nombre de modèles possibles. C'est sous forme d'une matrice qu'on peut représenter un courant de marchandises, ou venant de, ou aboutissant à, l'un des secteurs de l'industrie ou de l'économie. L'auteur discute les trois questions principales, qui résultent de ce calcul de matrices, à savoir, l'établissement de la période à envisager, la délimitation des secteurs individuels, et la manière de mesurer les courants de marchandises.

Ensuite, il étudie d'abord les modèles input-output statiques, et particulièrement le modèle statique de Leontief, dont on se sert le plus souvent pour l'analyse des relations interindustrielles, sous forme du modèle fermé ainsi que du modèle ouvert. Puis, il s'occupe des modèles dynamiques des relations interindustrielles, qui sont de nature plus générale que les modèles statiques, et qui permettent même des
prognoses du développement économique futur. L'auteur étudie deux espèces de modèles dynamiques plus en détail, à savoir, un modèle d'investissement et un modèle avec un «lag» entre output et input.

Resumen: Sobre los modelos para el examen del enlace económico de la industria. — El autor trata los modelos algebraicos que son empleados en el examen de las relaciones interindustriales. Es por estos modelos que se quieren explicar cuantitativamente las corrientes de bienes y de servicios entre distintos sectores de la economía o de la industria. A pesar de esta limitación de ello resulta un gran número de modelos posibles. Se puede representar en forma de una matriz la corriente de bienes en respectivo de uno de los distintos sectores de la economía o de la industria. El autor discute las tres cuestiones principales que resultan de este cálculo de matrices, a saber la fijación del periodo, la delimitación de los distintos sectores y la medida de las corrientes de bienes. A continuación trata los modelos estáticos de input-output, sobre todo el modelo estático de Leontief que es empleado lo más frecuentemente para el análisis de las relaciones interindustriales, en forma del modelo cerrado y abierto. Sigue la discusión de modelos dinámicos de las relaciones interindustriales que son de naturaleza más general que los modelos estáticos y que facilitan también pronósticos sobre el desarrollo económico. El autor trata más extensamente dos especies de estos modelos dinámicos, a saber un modelo de investición y otro con un «lag» entre output e input.

Riassunto: Suimodelliperl’esame dell’intrecciamento economico dell’industria. — L’autore tratta i modelli algebraici che sono usati nell’esame dei rapporti interindustriali. È per mezzo di questi modelli che si vuole esplicare quantitativamente le correnti di beni e di servizi fra i singoli settori dell’economia o dell’industria. Malgrado questa limitazione da ciò risulta un gran numero di modelli possibili. Si può rappresentare in forma di una matrice la corrente di beni in rispettivamente da uno dei differenti settori dell’economia o dell’industria. L’autore discute le tre questioni principali che risultano da questo calcolo di matrici cioè la fissazione del periodo, la demarcazione dei differenti settori e la misurazione delle correnti di beni. In seguito di ciò tratta i modelli statici input-output, soprattutto il modello statico del Leontief che è usato per lo più per l’analisi dei rapporti interindustriali, in forma del modello chiuso e aperto. Segue la discussione di modelli dinamici dei rapporti interindustriali che sono di natura più generale che i modelli statici e che facilitano anche delle pro- gnosi sullo sviluppo economico. L’autore tratta più a lungo due specie di questi modelli dinamici cioè un modello d’investizione e altro con un «lag» fra output e input.